

PARTICLE BASIS OF FEIGIN-STOYANOVSKY'S TYPE SUBSPACES OF LEVEL 1 $\tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$ -MODULES

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ABSTRACT. We construct particle basis for Feigin-Stoyanovsky's type subspaces of level 1 standard $\tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$ -modules. From the description we obtain character formulas.

1. INTRODUCTION

A problem of finding a monomial basis of a standard module is part of the Lepowsky-Wilson's program of studying Rogers-Ramanujan type identities through representation theory of affine Lie algebras ([LW], [LP], [MP]). Description of basis was used to obtain graded dimension of these modules, which gave the sum-side in Rogers-Ramanujan-type identities.

B. Feigin and A. Stoyanovsky initiated another approach to Rogers-Ramanujan type identities by considering what they called a principal subspace of a standard $\tilde{\mathfrak{sl}}_2(\mathbb{C})$ -module ([FS]). These subspaces were further studied by G. Georgiev ([G]), C. Calinescu, S. Capparelli, J. Lepowsky and A. Milas ([CLM1,2], [CalLM1,2,3], [C1,2]), C. Sadowski ([S1,2]), E. Ardonne, R. Kedem and M. Stone ([AKS]).

Another type of principal subspace, called Feigin-Stoyanovsky's type subspace, was introduced and studied by M. Primc who constructed a basis of this subspace and from it he obtained basis of the whole standard module ([P1,2]). For $\tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$, these bases were parameterized by $(k, \ell + 1)$ -admissible configurations ([FJLMM]), combinatorial objects introduced and further studied by Feigin et al. in [FJLMM] and [FJMMT], where bosonic and fermionic formulas for characters were obtained. Primc and M. Jerković obtained fermionic formulas for characters of standard $\tilde{\mathfrak{sl}}_3(\mathbb{C})$ -modules by using intertwining operators and admissible configurations ([J2]), or by using quasi-particle bases [JP]). In our previous work ([T]), we have used $(1, \ell + 1)$ -admissible configurations to combinatorially obtain character formulas for Feigin-Stoyanovsky's type subspaces of level 1 standard $\tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$ -modules. In this note we use an approach similar to Georgiev and Jerković and Primc to construct particle basis for Feigin-Stoyanovsky's type subspaces of level 1 standard $\tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$ -modules. From this description we immediately obtain character formulas.

2. AFFINE LIE ALGEBRA $\hat{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$

Let $\mathfrak{g} = \mathfrak{sl}_{\ell+1}(\mathbb{C})$ be a simple finite-dimensional Lie algebra of type A_ℓ . Fix a Cartan subalgebra $\mathfrak{h} \subset \mathfrak{g}$ and denote by R the corresponding root system; \mathfrak{g} has a root decomposition $\mathfrak{g} = \mathfrak{h} \oplus \coprod_{\alpha \in R} \mathfrak{g}_\alpha$. Let $\Pi = \{\alpha_1, \dots, \alpha_\ell\}$ be a basis of the root system R , and let $\{\omega_1, \dots, \omega_\ell\}$ be the corresponding set of fundamental weights, $\langle \omega_i, \alpha_j \rangle = \delta_{ij}$. Set $\omega_0 = 0$ for convenience. Let $\langle \cdot, \cdot \rangle$ be a normalized invariant

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bilinear form on \mathfrak{g} ; we identify \mathfrak{h} with \mathfrak{h}^* via $\langle \cdot, \cdot \rangle$. Denote by Q the root lattice, and by P the weight lattice of \mathfrak{g} . Also for each root $\alpha \in R$ fix a root vector $x_\alpha \in \mathfrak{g}_\alpha$.

Let $\tilde{\mathfrak{g}}$ be the associated untwisted affine Lie algebra $([\mathbf{K}])$,

$$\tilde{\mathfrak{g}} = \mathfrak{g} \otimes \mathbb{C}[t, t^{-1}] \oplus \mathbb{C}c \oplus \mathbb{C}d.$$

Denote by $x(m) = x \otimes t^m$ for $x \in \mathfrak{g}$, $m \in \mathbb{Z}$, and define formal Laurent series $x(z) = \sum_{m \in \mathbb{Z}} x(m) z^{-m-1}$. Denote by $\Lambda_0, \dots, \Lambda_\ell$ fundamental weights for $\tilde{\mathfrak{g}}$.

Fix a minuscule weight $\omega = \omega_\ell$ and set

$$\Gamma = \{ \alpha \in R \mid \langle \alpha, \omega \rangle = 1 \} = \{ \alpha_i + \dots + \alpha_\ell \mid i = 1, \dots, \ell \}.$$

Denote by $\gamma_i = \alpha_i + \dots + \alpha_\ell$. Then

$$\mathfrak{g} = \mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1, \quad \mathfrak{g}_0 = \mathfrak{h} \oplus \sum_{\langle \alpha, \omega \rangle = 0} \mathfrak{g}_\alpha, \quad \mathfrak{g}_{\pm 1} = \sum_{\alpha \in \pm \Gamma} \mathfrak{g}_\alpha,$$

is a \mathbb{Z} -gradation of \mathfrak{g} . Subalgebras \mathfrak{g}_1 and \mathfrak{g}_{-1} are commutative.

The \mathbb{Z} -gradation of \mathfrak{g} gives the \mathbb{Z} -gradation of the affine Lie algebra $\tilde{\mathfrak{g}}$:

$$\tilde{\mathfrak{g}} = \tilde{\mathfrak{g}}_{-1} + \tilde{\mathfrak{g}}_0 + \tilde{\mathfrak{g}}_1, \quad \tilde{\mathfrak{g}}_0 = \mathfrak{g}_0 \otimes \mathbb{C}[t, t^{-1}] \oplus \mathbb{C}c \oplus \mathbb{C}d, \quad \tilde{\mathfrak{g}}_{\pm 1} = \mathfrak{g}_{\pm 1} \otimes \mathbb{C}[t, t^{-1}].$$

Again, $\tilde{\mathfrak{g}}_{-1}$ and $\tilde{\mathfrak{g}}_1$ are commutative subalgebras. We will call elements $\gamma \in \Gamma$ *colors* and we will say that $x_\gamma(-m)$ is an element of *color* γ and *degree* m .

Let $L(\Lambda_r)$ be a standard (i.e. integrable highest weight) $\tilde{\mathfrak{g}}$ -module of level 1. Denote by v_r the highest weight vector of $L(\Lambda_r)$. Define a *Feigin-Stoyanovsky's type subspace*

$$W(\Lambda_r) = U(\tilde{\mathfrak{g}}_1) \cdot v_r \subset L(\Lambda_r).$$

By Poincaré-Birkhoff-Witt theorem, we have a spanning set of $W(\Lambda_r)$ consisting of monomial vectors

$$(1) \quad \{bv_r \mid b = x_\ell(-m_{\ell, n_\ell}) \cdots x_1(-m_{1, n_1}) \cdots x_1(-m_{1, 1}), m_{i, j+1} \geq m_{i, j} > 0, n_i \geq 0\},$$

where we write x_i instead of x_{γ_i} , for short.

3. VOA CONSTRUCTION

We briefly recall the vertex operator algebra construction of standard $\tilde{\mathfrak{g}}$ -modules $L(\Lambda_r)$ from [FK], [S]. For details and notation we turn the reader to [FLM], [DL] and [LL].

Consider tensor products $V_P = M(1) \otimes \mathbb{C}[P]$ and $V_Q = M(1) \otimes \mathbb{C}[Q]$, where $M(1)$ is the Fock space for the Heisenberg subalgebra $\hat{\mathfrak{h}}_{\mathbb{Z}} = \sum_{n \in \mathbb{Z} \setminus \{0\}} \mathfrak{h} \otimes t^n \oplus \mathbb{C}c$, and $\mathbb{C}[P]$ and $\mathbb{C}[Q]$ are group algebras of the weight and root lattice with bases consisting of $\{e^\lambda \mid \lambda \in P\}$, and $\{e^\alpha \mid \alpha \in Q\}$, respectively. We identify $\mathbb{C}[P]$ with $1 \otimes \mathbb{C}[P] \subset V_P$.

Space V_Q has a structure of vertex operator algebra and V_P is a module for this algebra:

$$(2) \quad Y(e^\lambda, z) = E^-(\lambda, z)E^+(\lambda, z) \otimes e^\lambda z^\lambda \epsilon(\lambda, \cdot),$$

where $E^\pm(\lambda, z) = \exp\left(\sum_{m \geq 1} \lambda(\pm m) z^{\mp m} / \pm m\right)$, e^λ is a multiplication operator, $z^\lambda \cdot e^\mu = e^\mu z^{\langle \lambda, \mu \rangle}$ and $\epsilon(\cdot, \cdot)$ is a 2-cocycle (cf. [FLM]).

By using vertex operators, one can define the structure of $\tilde{\mathfrak{g}}$ -module on V_P by setting $x_\alpha(z) = Y(e^\alpha, z)$ for $\alpha \in R$. This gives $V_Q \simeq L(\Lambda_0)$ and $V_Q e^{\omega_r} \simeq L(\Lambda_r)$, with highest weight vectors $v_0 = 1$ and $v_r = e^{\omega_r}$, and $V_P \cong L(\Lambda_0) \oplus \dots \oplus L(\Lambda_\ell)$.

From vertex operator formula (2) one easily obtains the following relations on $L(\Lambda_r)$

$$\begin{aligned} (3) \quad & x_i^2(z) = 0, \quad 1 \leq i \leq \ell, \\ (4) \quad & x_i(z)x_j(z) = 0, \quad 1 \leq i < j \leq \ell, \\ (5) \quad & x_i(m)v_r = 0, \quad m \geq -\delta_{i \leq r}, \\ (6) \quad & x_r(-1)v_{r-1} = Ce^{\omega_\ell + \omega_r} = Ce^{\omega_\ell}v_r, \end{aligned}$$

for some $C \in \mathbb{C}^\times$. Here, $\delta_{i \leq j}$ is 1 if $i \leq j$, 0 otherwise.

For the proof of linear independence we will be using certain coefficients of intertwining operators

$$\mathcal{Y}(e^\lambda, z) = Y(e^\lambda, z)e^{i\pi\lambda}c(\cdot, \lambda),$$

for $\lambda \in P$, where $c(\cdot, \lambda)$ is a commutator map (cf. [DL]). Let $\lambda_i = \omega_i - \omega_{i-1}$ for $i = 1, \dots, \ell$. From Jacobi identity ([DL]) we see that operators $\mathcal{Y}(e^{\lambda_i}, z)$ commute with the action of $\tilde{\mathfrak{g}}_1$. Define the following coefficients of intertwining operators (cf. [P3])

$$[i] = \text{Res } z^{-1-\langle \lambda_i, \omega_{i-1} \rangle} \mathcal{Y}(e^{\lambda_i}, z),$$

for $i = 1, \dots, \ell$. From (2), it follows (cf. [P3])

$$(7) \quad [i]v_{i-1} = Cv_i,$$

for some $C \in \mathbb{C}^\times$.

We will also be using simple current operators e^{ω_i} , $i = 1, \dots, \ell$. For $\alpha \in R$ and $\lambda \in P$ from (2) we get the following commutation relation

$$x_\alpha(z)e^\lambda = \epsilon(\alpha, \lambda)z^{\langle \alpha, \lambda \rangle}e^\lambda x_\alpha(z).$$

By comparing coefficients, we get

$$x_\alpha(m)e^\lambda = \epsilon(\alpha, \lambda)e^\lambda x_\alpha(m + \langle \alpha, \lambda \rangle).$$

In particular, for $\alpha = \gamma_i$ and $\lambda = \omega_j$, we get

$$(8) \quad x_i(m)e^{\omega_j} = \epsilon(\gamma_i, \omega_j)e^{\omega_j}x_i(m + \delta_{i \leq j}).$$

4. BASIS OF $W(\Lambda_r)$

To reduce the spanning set (1) and to prove linear independence, we need a linear order on monomials. Define a linear order $x_i(n) < x_j(m)$ if either $i > j$ or $i = j$ and $n < m$. We assume that in all monomials factors are sorted descendingly from right to left, like in (1). We compare two monomials b_1 and b_2 by comparing their factors from right to left (reverse lexicographic order): $b_1 < b_2$ if either $b_2 = bb_1$ or $b_1 = b'_1 x_i(n)b$, $b_2 = b'_2 x_j(m)b$ and $x_i(n) < x_j(m)$, for some monomials b, b_1, b_2 . This linear order is compatible with multiplication: if $b > c$, then $ab > ac$.

For a monomial $b = x_\ell(-m_{\ell, n_\ell}) \cdots x_\ell(-m_{\ell, 1}) \cdots x_1(-m_{1, n_1}) \cdots x_1(-m_{1, 1})$ define its *degree*, *weight* and *length* by $d(b) = m_{\ell, n_\ell} + \cdots + m_{\ell, 1} + \cdots + m_{1, n_1} + \cdots + m_{1, 1}$, $w(b) = n_1 \gamma_1 + \cdots + n_\ell \gamma_\ell$ and $l(b) = n_1 + \cdots + n_\ell$.

Theorem 1. *A spanning set of $W(\Lambda_r)$ is given by the set of monomial vectors (1) satisfying initial conditions*

$$(9) \quad m_{i, n} \geq 1 + \sum_{i < j} n_i + \delta_{j \leq r}$$

and difference conditions

$$(10) \quad m_{i, n+1} \geq m_{i, n} + 2, \quad 1 \leq n \leq n_i - 1.$$

Proof: Difference conditions follow from (3): Assume that b doesn't satisfy (10). Then $b = b'x_j(-m)x_j(-m')$, for some monomial b' and $m' \leq m \leq m' + 1$. By (3) and (5), on $W(\Lambda_r)$ we have

$x_j(-m)x_j(-m') = C_1x_j(-m-1)x_j(-m'+1) + \dots + C_{m'-1}x_j(-m-m'+1)x_j(-1)$, for some $C_i \in \mathbb{C}^\times$. Multiply this by b' to obtain b expressed as a linear combination of greater monomials of the same degree and weight.

Now assume that b doesn't satisfy (9); let $b = b_2x_j(-m)b_1$ where b_1 contains all factors of colors $\gamma_1, \dots, \gamma_{j-1}$ and

$$m < 1 + \sum_{i < j} n_i + \delta_{j \leq r}.$$

We will prove that b can be expressed in terms of greater monomials of the same degree and weight. The proof is done by induction on the length $l(b_1) = \sum_{i < j} n_i$. If $l(b_1) = 0$, then (5) gives $x_j(-m)v_r = 0$. Now, assume that all monomials a with $l(a_1) < l(b_1)$ can be expressed with greater monomials of the same degree and weight. We can also assume that $m = \sum_{i < j} n_i + \delta_{j \leq r}$. Let $x_k(-n)$ be the smallest factor in b_1 ; $b_1 = x_k(-n)b'_1$. By (4) and (5) we have

$$(11) \quad \begin{aligned} x_j(-m)x_k(-n) &= C_{1+\delta_{j \leq r}}x_j(-1-\delta_{j \leq r})x_k(-n-m+1+\delta_{j \leq r}) + \dots \\ &\quad + C_{m-1}x_j(-m+1)x_k(-n-1) + C_{m+1}x_j(-m-1)x_k(-n+1) + \dots \end{aligned}$$

for some $C_i \in \mathbb{C}$. Multiply this with $b_2b'_1$ and obtain

$$(12) \quad \begin{aligned} b &= b_2x_j(-1-\delta_{j \leq r})x_k(-n-m+1+\delta_{j \leq r})b'_1 + \dots \\ &\quad + b_2x_j(-m+1)x_k(-n-1)b'_1 + b_2x_j(-m-1)x_k(-n+1)b'_1 + \dots \end{aligned}$$

On the right-hand side we have monomials

$$b_2x_j(-m-1)x_k(-n+1)b'_1, b_2x_j(-m-2)x_k(-n+2)b'_1, \dots$$

which are greater than b . But we also have the first few monomials

$$(13) \quad b_2x_j(-1-\delta_{j \leq r})x_k(-n-m+1+\delta_{j \leq r})b'_1, \dots, b_2x_j(-m+1)x_k(-n-1)b'_1.$$

Consider their factors $x_j(-1-\delta_{j \leq r})b'_1, \dots, x_j(-m+1)b'_1$. By the inductive assumption, they can be expressed as linear combinations of greater monomials of the same degree and weight. Then it is obvious that by multiplying these linear expressions by $b_2x_k(-n-m+1+\delta_{j \leq r}), \dots, b_2x_k(-n-1)$ we obtain linear expressions for monomials in (13) in terms of greater monomials. Moreover, these monomials will also be greater than b . Hence, we have expressed b in terms of greater monomial. \square

Theorem 2. *A spanning set*

$$(14) \quad \mathcal{B} = \{bv_r | b \text{ satisfies (10) and (9)}\}$$

is a basis of $W(\Lambda_r)$.

Proof: Let $b \in \mathcal{B}$. We first prove a particular case: if

$$Cbv_r = 0$$

then $C = 0$. We prove this by induction on degree of b . Let $x_i(-n)$ be the greatest factor in b ; $b = b'x_i(-n)$.

If $i \leq r$ then, since $v_r = e^{\omega_r} = e^{\omega_r}v_0$, we have

$$Cbv_r = Cbe^{\omega_r}v_0 = e^{\omega_r}C'b''v_0 = 0,$$

where $C' \in \mathbb{C}^\times$ and b'' is obtained from b by decreasing degrees of factors of color $\gamma_1, \dots, \gamma_r$ by 1 (see (8)). Since e^{ω_r} is injective, $C'b''v_0 = 0$. Monomial b'' satisfies difference and initial conditions for $W(\Lambda_0)$ and it is of a smaller degree than b . By the inductive assumption, we conclude that $C = 0$.

If $i > r$ and $n > 1$ then use operators $[i][i-1] \cdots [r+1]$ to obtain

$$Cbv_i = 0.$$

Then, by (8),

$$Cbv_i = Cbe^{\omega_i}v_0 = e^{\omega_i}C'Cb''v_0 = 0,$$

where $C' \in \mathbb{C}^\times$ and b'' is obtained from b by decreasing degrees of factors of color γ_i by 1 (see (7) and (8)). Again, b'' satisfies difference and initial conditions for $W(\Lambda_0)$ and it is of a smaller degree than b . By induction, we conclude that $C = 0$.

If $i > r$ and $n = 1$ then use operators $[i-1] \cdots [r+1]$ to obtain

$$Cbv_{i-1} = 0$$

(see (7)). By (6) and (8), we have

$$Cb'x_i(-1)v_i = C'Cb'e^{\omega_\ell}v_i = e^{\omega_\ell}C''Cb''v_i = 0,$$

where $C', C'' \in \mathbb{C}^\times$ and b'' is obtained from b' by decreasing degrees of all factors by 1. Monomial b'' satisfies difference and initial conditions for $W(\Lambda_i)$ and it is of a smaller degree than b . By induction, we conclude that $C = 0$.

Now turn to the general relation of linear dependence:

$$(15) \quad \sum_b C_b bv_r = 0.$$

Let b_{\min} be the smallest monomial in (15). We use the same operators as in the previous case to peel down $C_{b_{\min}}b_{\min}v_r$ to $C_{b_{\min}}v_j$, for some j . Note that operators that we used above at some point annihilate other monomial vectors in (15). We will get

$$C_{b_{\min}}v_j = 0$$

and we conclude $C_{b_{\min}} = 0$. We proceed inductively to conclude that all coefficients C_b in (15) are 0. \square

From this combinatorial description of basis of $W(\Lambda_r)$ we immediately obtain character formulas (cf. [J1], [T]): for $n_1, \dots, n_\ell \geq 0$, set $\alpha = n_1\gamma_1 + \cdots + n_\ell\gamma_\ell$ and $\chi_{W(\Lambda_r)}^\alpha(q) = \sum_i q^i \text{card} \{b \mid w(b) = \alpha, d(b) = i\}$

Corollary 3.

$$\chi_{W(\Lambda_r)}^\alpha(q) = \frac{q^{\sum_{i=1}^\ell n_i^2 + \sum_{1 \leq i < j \leq \ell} n_i n_j + \sum_{i=1}^r n_i}}{(q)_{n_1}(q)_{n_2} \cdots (q)_{n_\ell}},$$

where $(q)_n = (1-q) \cdots (1-q^n)$.

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